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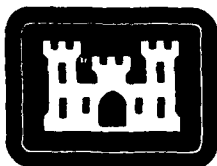
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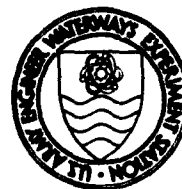


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# ADVECTIVE EFFECTS IN THE NUMERICAL SIMULATION OF LONG PERIOD LARGE AMPLITUDE WAVE BEHAVIOR

by

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September 1981

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20. ABSTRACT (continued).

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The primary objective of this investigation was the assessment of the role of advective terms in the numerical simulation of long period, large amplitude wave behavior. A major effort in accomplishing this objective was the development of an appropriate representation of the non-linear terms in the difference equations of motion. To this end, a stabilizing correction, double sweep implicit scheme was encoded into an existing and extensively used hydrodynamic model, WIFM. The scheme manifested a high degree of stability in various numerical flume tests.

In previous applications WIFM was used strictly as an inland flooding model. Efforts within this research investigation as well as concurrent studies permitted extension of WIFM to include continental shelf simulation. Comparisons with a recognized open coast surge model were made, and results indicated WIFM is appropriate for treating the global problem (shelf dynamics and coastal flooding) in a single grid.

Simulations for the Louisiana coastline under storm attack by Hurricane Betsy were made with and without including the non-linear terms' effect. Results showed that non-linear term effect on surface elevation was minimal but more noticeable at the coast near landfall. Effect on current patterns was more pronounced. Further testing is recommended in future U. S. Army surge investigations.

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## PREFACE

The study described herein was authorized under the In-House Laboratory Independent Research Program. All elements of the investigation were conducted at the U. S. Army Engineer Waterways Experiment Station (WES) during the period November 1977 to September 1981 in the Hydraulics Laboratory by Mr. H. L. Butler under the direction of Mr. H. B. Simmons, Chief, Hydraulics Laboratory, and Dr. R. W. Whalin, Chief, Wave Dynamics Division.

Numerical computations associated with this work were performed on the CYBER 176 and CRAY 1 computers located at the Los Alamos Scientific Laboratory and the Air Force Weapons Laboratory, Kirtland AFB, New Mexico.

Commanders and Directors of WES during the course of the investigation were COL John L. Cannon, CE, COL Nelson P. Conover, CE, and COL Tilford C. Creel, CE. Technical Director was Mr. F. R. Brown.

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## ABSTRACT

Numerical modeling of water-wave behavior has progressed rapidly in the last decade and is now generally recognized as a useful tool capable of providing solutions to many coastal problems. The U. S. Army, through various agencies, has sponsored development of two-dimensional numerical models for simulating long period wave behavior. Many of these models have included the advective terms in their formulation. Demonstrated in numerous papers throughout the literature, inclusion of advective effects can lead to instabilities in the solution. This fact was exemplified in work of Sloss (1972) in his attempt to include these effects in storm surge simulation.

The primary objective of this investigation is the assessment of the role of advective terms in the numerical simulation of long period, large amplitude wave behavior. A major effort in accomplishing this objective was the development of an appropriate representation of the non-linear terms in the difference equations of motion. To this end a stabilizing correction, double sweep implicit scheme was encoded into an existing and extensively used hydrodynamic model, WIFM. The scheme manifested a high degree of stability in various numerical flume tests.

In previous applications WIFM was used strictly as an inland flooding model. Efforts within this research investigation as well as concurrent studies permitted extension of WIFM to include continental shelf simulation. Comparisons to a recognized open coast surge model were made and results indicated WIFM was appropriate for treating the global problem (shelf dynamics and coastal flooding) in a single grid.

Simulations for the Louisiana coastline under storm attack by Hurricane Betsy were made with and without including the non-linear terms' effect. Results showed that non-linear term effect on surface elevation was minimal but more noticeable at the coast near landfall. Effect on current patterns was more pronounced. Further testing is recommended in future U. S. Army surge investigations.



## PART I: INTRODUCTION

### Background

1. Numerical modeling of water-wave behavior has progressed rapidly in the last decade and is now generally recognized as a useful tool capable of providing solutions to many coastal problems. The U. S. Army, through various agencies, has sponsored development of two-dimensional numerical models for simulating long-period wave behavior. Many of these models have included the advective terms in their formulation. Demonstrated in numerous papers throughout the literature, inclusion of advective effects can lead to instabilities in the solution. This fact was exemplified in work of Sloss (1972) in his attempt to include these effects in storm surge simulation.

2. The Corps of Engineers has had to address the problem of providing reliable estimates of estuarine circulation and coastal flooding from tides as well as large amplitude phenomena such as storm surge, tsunamis, and land-slide or explosion generated water waves. These simulations are used to make sound engineering decisions regarding the design, operation, and maintenance of various coastal projects. It is mandatory that the Army have numerical prediction models which can propagate long period, large amplitude waves into nearshore and overland regions. The success of base operations, combat maneuvers, and emergency aid to civil defense needs, are linked to accurate prediction of catastrophic water waves. The role of the advective terms under these conditions must be assessed to provide a more complete understanding of the associated hydrodynamics.

### Accuracy and Non-Linear Aspects of Difference Schemes

3. The influence of the time step on accuracy of the difference scheme is very important. It is characterized by the dimensionless quantity  $k = \frac{|c|\Delta t}{\Delta x}$  where  $c$  is the maximum wave speed and  $\Delta t$ ,  $\Delta x$  are

the time and grid step sizes, respectively. When using explicit schemes, linear stability investigations require  $k \leq 1$ . With implicit schemes, this restriction usually does not apply but accuracy does diminish with increasing  $k$ . Yet another phenomenon can occur, introduction of non-linear instabilities which totally destroy the solution. These instabilities have been shown (Kuipers and Vreugdenhil, 1973) to be directly related to inclusion of the advective terms in the difference equations. Omission of these terms makes the instabilities disappear, but also makes it impossible to compute circulation currents and horizontal eddies. Analysis shows that flow patterns from stable computations are disturbed by small vortices with a diameter the size of a grid cell. Vorticity is obviously transferred toward the smallest possible scale and is not properly removed. The advective terms are indispensable for accurate representation of vorticity transport. The question raised in this report is whether these terms are important in modeling long period, large amplitude waves, particularly, in catastrophic flood situations.

#### Objectives

4. The primary objective of this investigation is the assessment of the role of advective terms in the numerical simulation of long period, large amplitude wave behavior. An implicit coastal flooding model, WIFM (Butler, 1978), was used to arrive at this assessment. WIFM has been applied extensively (Butler, 1981) but the non-linear instability problem appeared in many of these applications. Thus, a major effort in this study was to develop an appropriate representation of the non-linear terms in the difference equations of motion.

5. Prior to this study WIFM had not been applied to an open-coast surge problem. This investigation, in conjunction with on-going cost-reimbursable studies, provided the impetus to extend WIFM to treat open-coast problems. The objectives were met by carrying out the tasks enumerated below:

a. Develop an accurate representation of the nonlinear advective terms in the WIFM algorithm.

b. Extend WIFM to treat open-coast surge problems and compare results of a WIFM simulation with a recognized open-coast model, SSURGE (Wanstrath, et al., 1976).

c. Assess advective effects in simulating the catastrophic flooding caused by Hurricane Betsy which struck the Louisiana coast in 1965.

## PART II: COMPUTATIONAL TECHNIQUES

### Equations of Motion

6. The equations of fluid flow used in WIFM are derived from the classical Navier-Stokes equations in a Cartesian coordinate system. By assuming vertical accelerations are small and the fluid is homogeneous and integrating the flow from sea bottom to water surface, the usual two-dimensional form of the equations of momentum and continuity are obtained. A major advantage of WIFM is the capability of applying a smoothly varying grid to a given study region permitting simulation of a complex landscape by locally increasing grid resolution and/or aligning coordinates along physical boundaries. For each direction, a piecewise reversible transformation which takes the form

$$x = a + b\alpha_1^c \quad (1)$$

where  $a$ ,  $b$ , and  $c$  are arbitrary constants, is independently used to map prototype or real space into computational space. Many stability problems commonly associated with variable grid schemes are eliminated via the continuity of the transformation procedure. The resulting equation of motion in  $\alpha$ -space can be written as

#### Momentum:

$$\begin{aligned} & u_t + \frac{1}{\mu_1} \frac{uu_{\alpha_1}}{\alpha_1} + \frac{1}{\mu_2} \frac{vu_{\alpha_2}}{\alpha_2} - fv \\ & + \frac{g}{\mu_1} (\eta - \eta_a)_{\alpha_1} + \frac{gu}{c^2 d} (u^2 + v^2)^{1/2} \\ & - \epsilon \left( \left( \frac{1}{\mu_1} \right)^2 \frac{u_{\alpha_1 \alpha_1}}{\alpha_1} + \frac{1}{\mu_1} \left( \frac{1}{\mu_1} \right)_{\alpha_1} \frac{u_{\alpha_1}}{\alpha_1} + \left( \frac{1}{\mu_2} \right)^2 \frac{u_{\alpha_2 \alpha_2}}{\alpha_2} \right. \\ & \left. + \frac{1}{\mu_2} \left( \frac{1}{\mu_2} \right)_{\alpha_2} \frac{u_{\alpha_2}}{\alpha_2} \right) + F_{\alpha_1} = 0 \end{aligned} \quad (2)$$

$$\begin{aligned}
v_t + \frac{1}{\mu_1} uv_{\alpha_1} + \frac{1}{\mu_2} vv_{\alpha_2} + fu + \frac{g}{\mu_2} (\eta - \eta_a)_{\alpha_2} \\
+ \frac{gv}{C^2 d} (u^2 + v^2)^{1/2} - \epsilon \left( \frac{1}{\mu_1} \right)^2 v_{\alpha_1 \alpha_1} + \frac{1}{\mu_1} \left( \frac{1}{\mu_1} \right)_{\alpha_1} v_{\alpha_1} \\
+ \frac{1}{\mu_2} \left( \frac{1}{\mu_2} \right)^2 v_{\alpha_2 \alpha_2} + \frac{1}{\mu_2} \left( \frac{1}{\mu_2} \right)_{\alpha_2} v_{\alpha_2} + F_{\alpha_2} = 0
\end{aligned} \quad (3)$$

Continuity:

$$\eta_t + \frac{1}{\mu_1} (du)_{\alpha_1} + \frac{1}{\mu_2} (dv)_{\alpha_2} = R \quad (4)$$

where

$$\mu_1 = \frac{\partial x}{\partial \alpha_1} \text{ and } \mu_2 = \frac{\partial y}{\partial \alpha_2}$$

and  $\eta$  is the water-surface elevation;  $\eta_a$  is the hydrostatic elevation corresponding to the atmospheric pressure anomaly;  $u$  and  $v$  are the vertically integrated velocities at time  $t$  in the  $\alpha_1$  and  $\alpha_2$  directions, respectively;  $d = \eta - h$  is the total water depth;  $h$  is the still-water elevation;  $f$  is the Coriolis parameter;  $C$  is the Chezy frictional coefficient;  $g$  is the acceleration of gravity;  $\epsilon$  is a generalized eddy viscosity coefficient;  $R$  represents the rate at which additional water is introduced into or taken from the system (for example, through rainfall and evaporation); and  $F_{\alpha_1}$  and  $F_{\alpha_2}$  are terms representing external forcing functions such as wind stress in the  $\alpha_1$  and  $\alpha_2$  directions. Quantities  $\mu_1$  and  $\mu_2$  define the stretching of the regular-spaced computational grid in  $\alpha$ -space to approximate a study region in real space. Directions  $\alpha_1$  and  $\alpha_2$  correspond to  $x$  and  $y$ , respectively. The single underlined terms are referred to as the advective inertia terms and the doubly underlined terms as the horizontal diffusion or eddy-viscosity terms.

### Finite Difference Formulation

7. The differential equations (Eqs. 2-4) are to be approximated by difference equations. Various solution schemes, including implicit and explicit formulations, could be used. WIFM permits a selection of difference formulations, but this report will concentrate on an alternating direction technique (ADI).

8. Prior to the subject study WIFM employed a typical ADI scheme (Butler, 1978) similar to Leendertse (1970). The problem with applying this procedure was maintaining stability when the advective inertia terms were included in the solution algorithm, particularly in storm surge or long period, large amplitude wave simulation. Weare (1976) indicated that the problem lay in the differencing techniques used, namely, in approximating the advective terms with expressions not centered in time. As a cure for this problem Weare (1979) introduced methods of developing alternate ADI solution schemes which did treat all terms centered in time and space. In particular, he suggested a stabilizing correction scheme (SC scheme) employing three full-time levels. The details of this development can be found in a paper by Butler (1980) and in a report (Butler, 1981) documenting the WIFM model.

9. If the linearized equations of motion are written in matrix form, one obtains

$$U_t + AU_x + BU_y = 0 \quad (5)$$

where

$$U = \begin{pmatrix} \eta \\ u \\ v \end{pmatrix}, \quad A = \begin{pmatrix} 0 & d & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & d \\ 0 & 0 & 0 \\ g & 0 & 0 \end{pmatrix}$$

The SC scheme for solving Equation 5 is

$$(1 + \lambda_x) U^* = (1 - \lambda_x - 2\lambda_y) U^{k-1} \quad (6)$$

$$(1 + \lambda_y) U^{k+1} = U^* + \lambda_y U^k \quad (7)$$

where

$$\lambda_x = \frac{1}{2} \frac{\Delta t}{\Delta x} A \delta_x \quad \text{and} \quad \lambda_y = \frac{1}{2} \frac{\Delta t}{\Delta y} B \delta_y$$

The quantities  $\delta_x$  and  $\delta_y$  are centered difference operators and superscript  $k$  counts time levels. The starred quantities can be considered approximate values for corresponding variables at the  $(k+1)$  time level.

10. The first step in each procedure is carried out by sweeping the grid in the  $x$  direction, and the second step is computed by sweeping in the  $y$  direction. Completing both sweeps constitutes a full time step, advancing the solution from the  $k$ th time level to the  $(k+1)$  time level. The form of the difference equations for the  $x$ -sweep is given by

$$\frac{1}{2\Delta t} (\eta^* - \eta^{k-1}) + \frac{1}{2\Delta x} \delta_x (u^* + u^{k-1}) + \frac{1}{\Delta y} \delta_y (v^{k-1}) = 0 \quad (8)$$

$$\frac{1}{2\Delta t} (u^* - u^{k-1}) + \frac{g}{2\Delta x} \delta_x (\eta^* + \eta^{k-1}) = 0 \quad (9)$$

$$\frac{1}{2\Delta t} (v^* - v^{k-1}) + \frac{g}{\Delta y} \delta_y (\eta^{k-1}) = 0 \quad (10)$$

and the  $y$ -sweep by

$$\frac{1}{2\Delta t} (\eta^{k+1} - \eta^*) + \frac{1}{2\Delta y} \delta_y (v^{k+1} - v^{k-1}) = 0 \quad (11)$$

$$u^{k+1} = u^* \quad (12)$$

$$\frac{1}{2\Delta t} (v^{k+1} - v^*) + \frac{g}{2\Delta y} \delta_y (\eta^{k+1} - \eta^{k-1}) = 0 \quad (13)$$

Noting that  $v^*$  in Equation 10 is only a function of previously computed variables at the  $(k-1)$  time level, the above equations can be simplified to give

x-sweep

$$\frac{1}{2\Delta t} (\eta^* - \eta^{k-1}) + \frac{1}{2\Delta x} \delta_x (u^{k+1}_d + u^{k-1}_d) + \frac{1}{\Delta y} \delta_y (v^{k-1}_d) = 0 \quad (14)$$

$$\frac{1}{2\Delta t} (u^{k+1} - u^{k-1}) + \frac{g}{2\Delta x} \delta_x (\eta^* + \eta^{k-1}) = 0 \quad (15)$$

y-sweep

$$\frac{1}{2\Delta t} (\eta^{k+1} - \eta^*) + \frac{1}{2\Delta y} \delta_y (v^{k+1}_d - v^{k-1}_d) = 0 \quad (16)$$

$$\frac{1}{2\Delta t} (v^{k+1} - v^{k-1}) + \frac{g}{2\Delta y} \delta_y (\eta^{k+1} + \eta^{k-1}) = 0 \quad (17)$$

11. Expanding the SC scheme to the full equations of motion, Equations 2-4, and defining the appropriate variables on each grid cell in a space-staggered fashion as depicted in Figure 1,

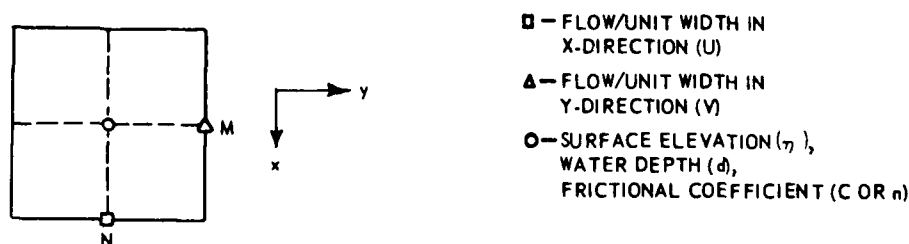


Figure 1. Computational Cell Definition

difference equations for the x-sweep (along a grid cell column parallel to the x-axis) can be written as



$$\begin{aligned} \frac{1}{2\Delta t} (\eta^* - \eta^{k-1}) + \frac{1}{2\mu_1\Delta\alpha_1} [\delta_{\alpha_1} (u^{k+1}\bar{d}^k + u^{k-1}\bar{d}^k)] \\ + \frac{1}{\mu_2\Delta\alpha_2} \delta_{\alpha_2} (v^{k-1}\bar{d}^k) = R \text{ at } (n,m) \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{1}{2\Delta t} (u^{k+1} - u^{k-1}) + \frac{1}{2\mu_1\Delta\alpha_1} u^k \delta_{2\alpha_1} (u^k) + \frac{1}{2\mu_2\Delta\alpha_2} \bar{v}^k \delta_{2\alpha_2} (u^k) \\ - f\bar{v}^k + \frac{g}{2\mu_1\Delta\alpha_1} [\eta^* + \eta^{k-1} - 2\eta_a^k] \\ + \frac{g}{(\bar{c}^2\bar{d})^k} u^{k+1} [(u^{k-1})^2 + (\bar{v}^{k-1})^2]^{1/2} - \epsilon [\frac{1}{(\mu_1\Delta\alpha_1)^2} \delta_{\alpha_1\alpha_1} (u^k) \\ + \frac{1}{(\mu_2\Delta\alpha_2)^2} \delta_{\alpha_2\alpha_2} (u^k) + \frac{1}{2\mu_1\Delta\alpha_1} \delta_{\alpha_1} (\frac{1}{\mu_1}) \delta_{2\alpha_1} (u^k) \\ + \frac{1}{2\mu_2\Delta\alpha_2} \delta_{\alpha_2} (\frac{1}{\mu_2}) \delta_{2\alpha_2} (u^k)] + F_{\alpha_1}^k = 0 \text{ } (n, m + \frac{1}{2}) \end{aligned} \quad (19)$$

In these expressions, a single bar represents a two-point average and a double bar a four-point average. The subscripts m and n correspond to spatial locations and superscript k to time levels. The difference operator  $\delta_\alpha$  is defined as

$$\delta_\alpha(Z) = Z_{\alpha+1/2} - Z_{\alpha-1/2}$$

for any variable Z.

12. Applying these equations at each grid cell in a given column results in a system of linear algebraic equations whose coefficient matrix is tridiagonal. The y-sweep is formulated in an analogous manner.

### Non-Linear Effects

13. Since the existence of the non-linear instabilities in previously applied difference schemes were shown to stem from the imperfect time-centering of difference representations of the non-linear terms (Weare, 1976), a fully time-centered scheme was adopted and encoded into WIFM. The horizontal diffusion terms also were included in the difference scheme to contribute to numerical stability (Vreugdenhil, 1973). These terms, strictly speaking, should be included in the momentum equations. Vreugdenhil demonstrates that such terms actually are representations of the effective stress terms usually neglected. As discussed in following paragraphs tests were made to determine effects of each term.

### Extension To Open-Coast Treatment

14. In previous surge applications (Butler and Wanstrath, 1976); Butler, 1978) WIFM was used strictly as an inland flooding model. Seaward boundary conditions were obtained from open-coast surge models by specifying surface elevation at the boundaries. To extend WIFM's capabilities to include shelf simulation only treatments of two additional boundary conditions were added. These were a uniform flux boundary condition and an inverted barometer effect condition. Capability of using the inverted barometer effect throughout the grid for initial conditions also was added.

### PART III: TEST APPLICATIONS

#### Flume Tests

15. To test the new WIFM algorithm, including representation of both advective and horizontal diffusion terms, two simple flume tests were devised. Both tests used a constant depth flume of 10 ft\*. One test was for an expanding flow and the second for flow around a breakwater. Various cell sizes were employed to test spatial resolution effects. Two types of assumptions were made for approximating the non-linear terms at closed boundaries: linerization at the boundary (A1) or one-sided difference quotients to replace these terms (A2). Linerization introduces additional boundary conditions approximating free slip which may influence secondary phenomena.

16. A constant value for the eddy-viscosity coefficient,  $\epsilon$ , was assumed over the spatial regime. Various values for  $\epsilon$  were tested. Increasing  $\epsilon$  beyond physically justifiable limits seriously affected both surface elevation, but more particularly, the flow pattern. Figures 2 and 3 depict circulation patterns for an expanding flow for 480 time steps and assumptions A1 and A2, respectively. A uniform flow was imposed at the left-hand boundary. Figures 4 and 5 depict full development after 960 time steps. Boundary effects for assumption A1 are obvious when a steady state condition was reached (Figure 4).

17. A barrier was inserted in the flume and a uniform flow was again imposed at the left. Figures 6-12 display sample tests for varying spatial resolution. Eddys are developed behind the breakwater as expected. When the resolution was decreased to only 4 cells across the tank no eddy developed. Figures 11 and 12 indicate that even a short barrier extension of two cell lengths will produce an eddy. Assumption A1 and A2 produced almost identical results except for slight differences at the tip of the breakwater. No definitive conclusion can be made on which assumption is better, but assumption A2 is a more accurate model of the physics involved and thus should be used.

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\* Conversion factor for feet to metres: multiply by 0.3048

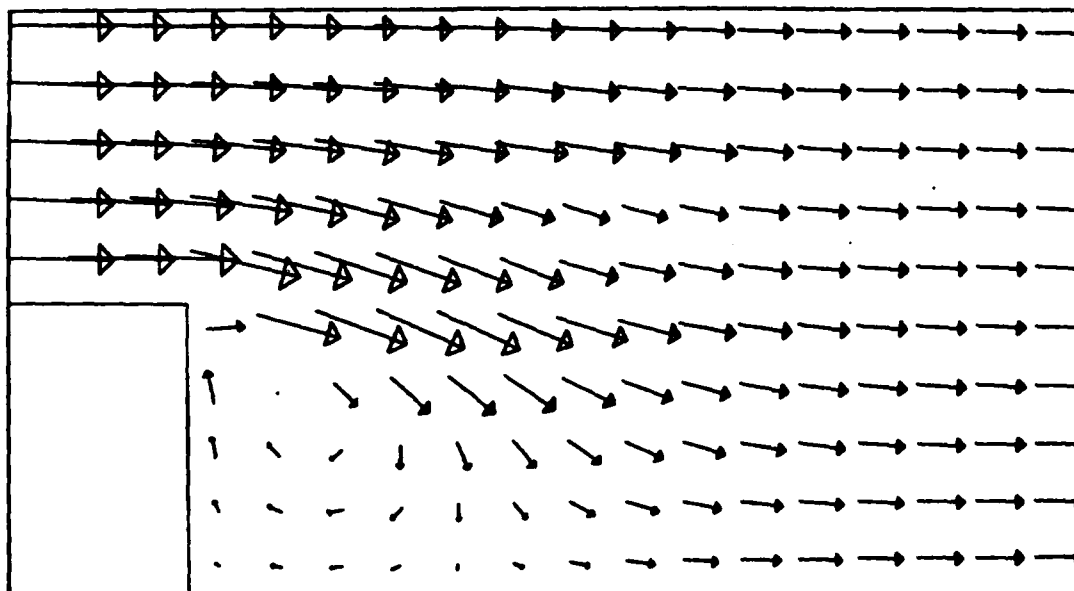


Figure 2. Expanding flow for assumption A1 after 480 time steps

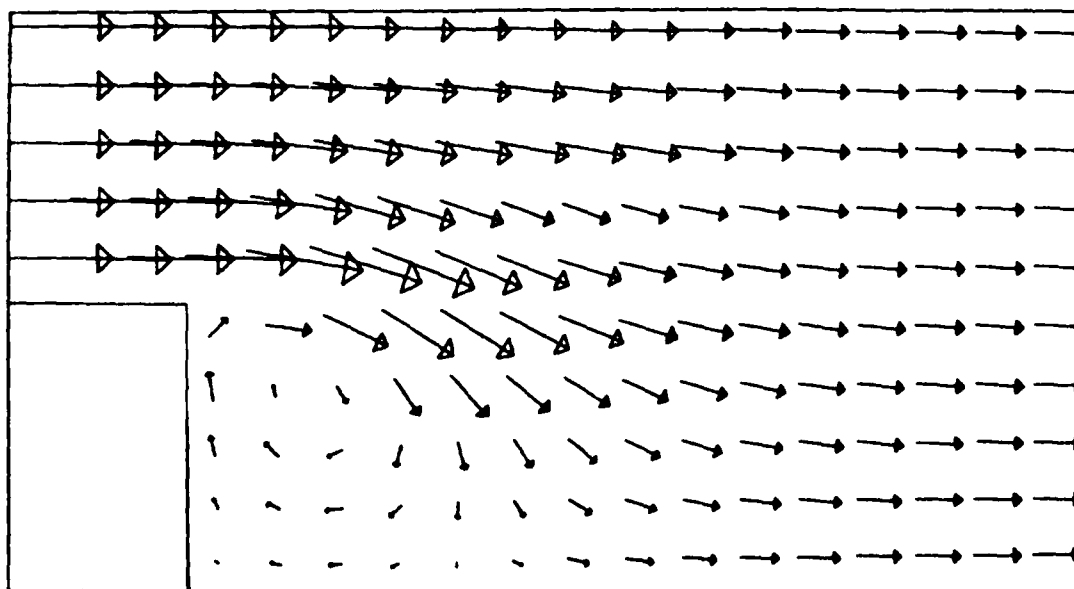


Figure 3. Expanding flow for assumption A2 after 480 time steps

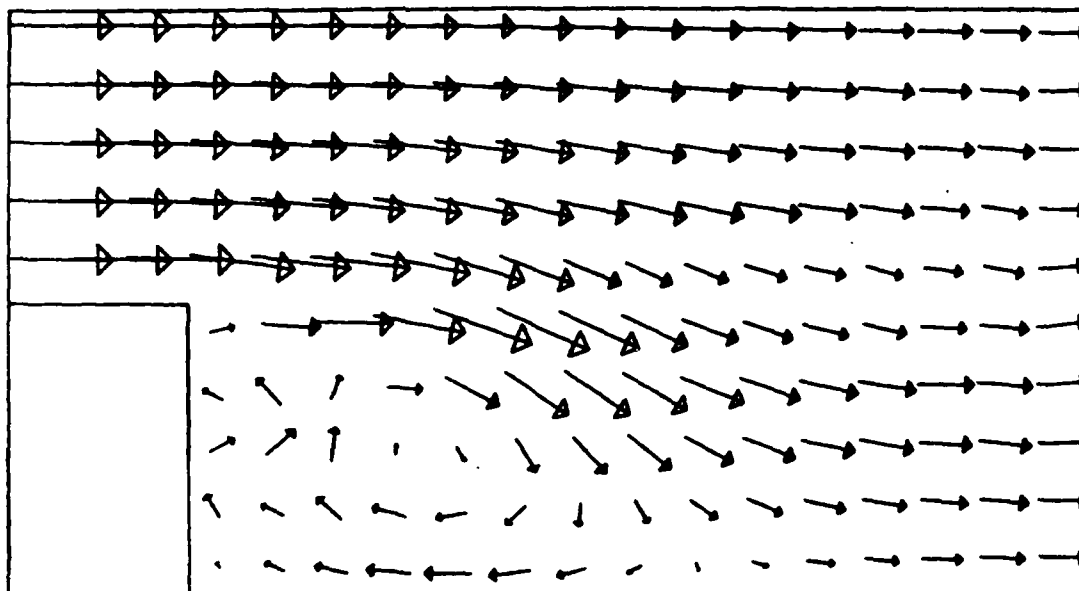


Figure 4. Expanding flow for assumption A1 after 960 time steps

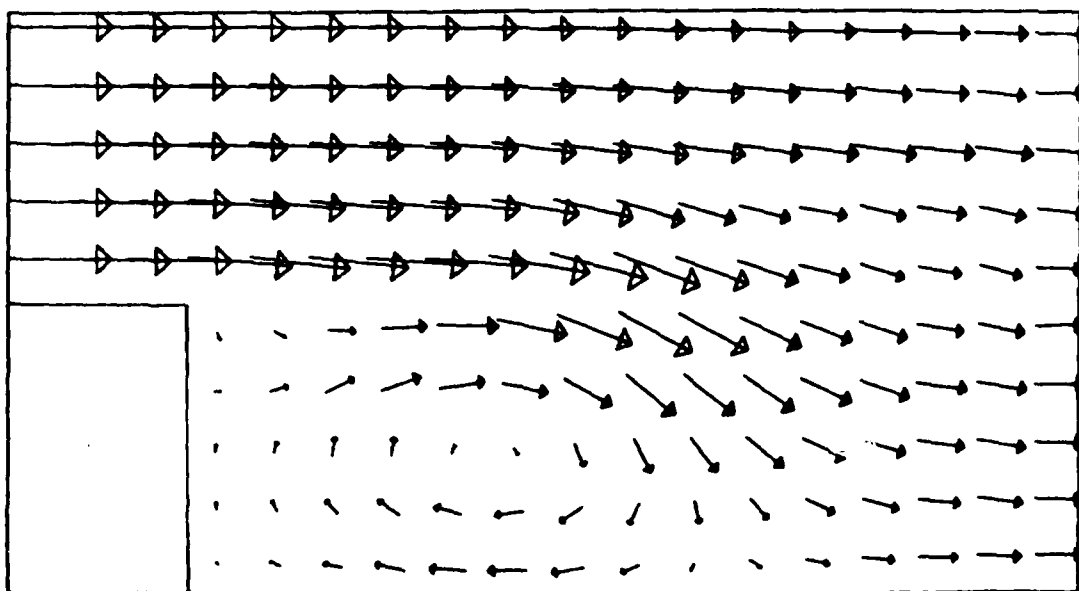


Figure 5. Expanding flow for assumption A2 after 960 time steps

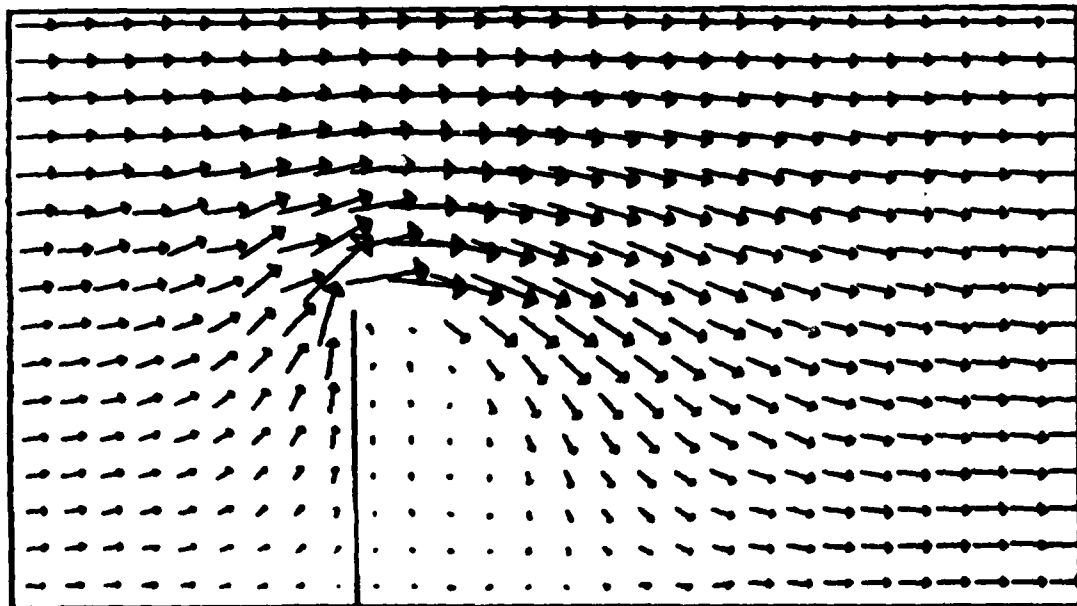


Figure 6. Flume barrier test with 16 cell resolution for assumption A2

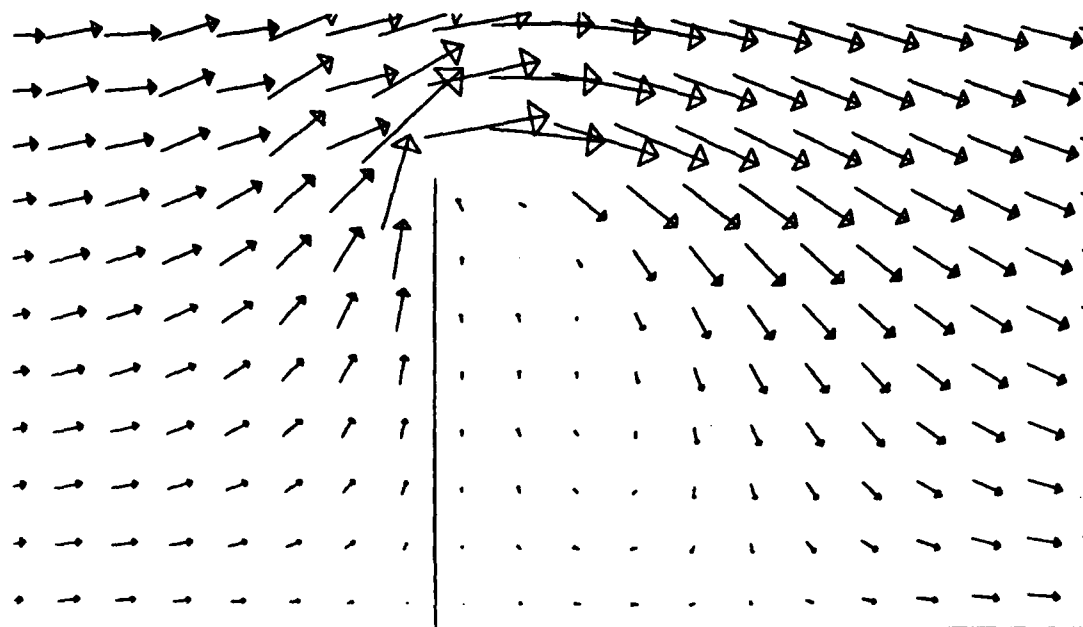


Figure 7. Enlargement of Figure 6 in area of barrier

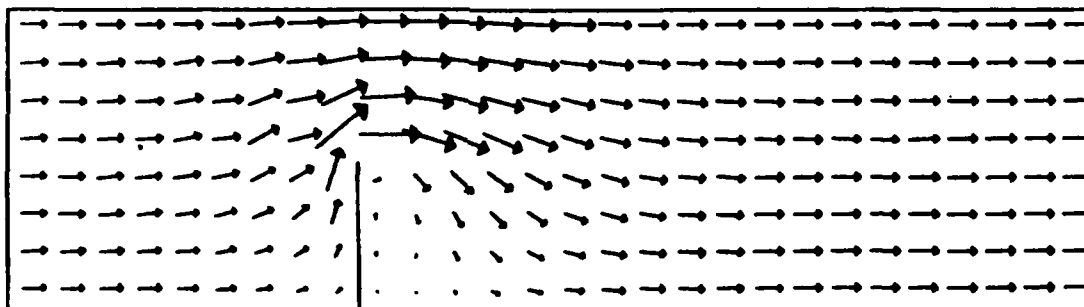


Figure 8. Flume barrier test with 8 cell resolution for assumption A2

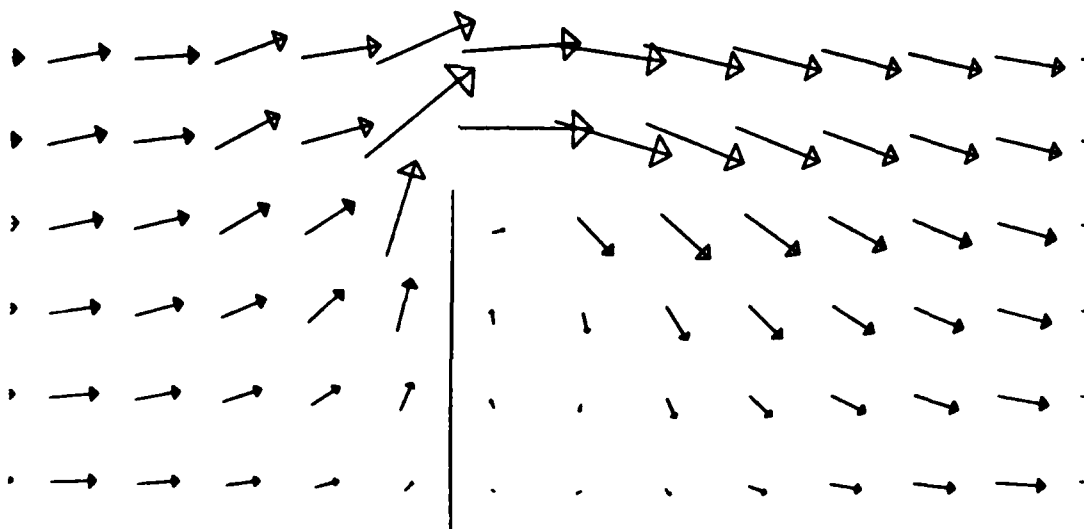


Figure 9. Enlargement of Figure 8 in area of barrier

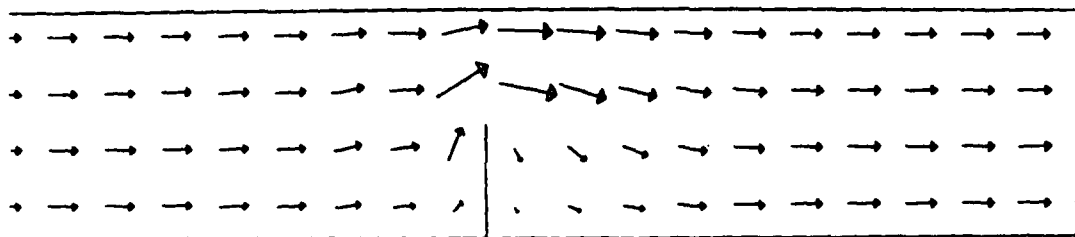


Figure 10. Flume barrier test with 4 cell resolution for assumption A2

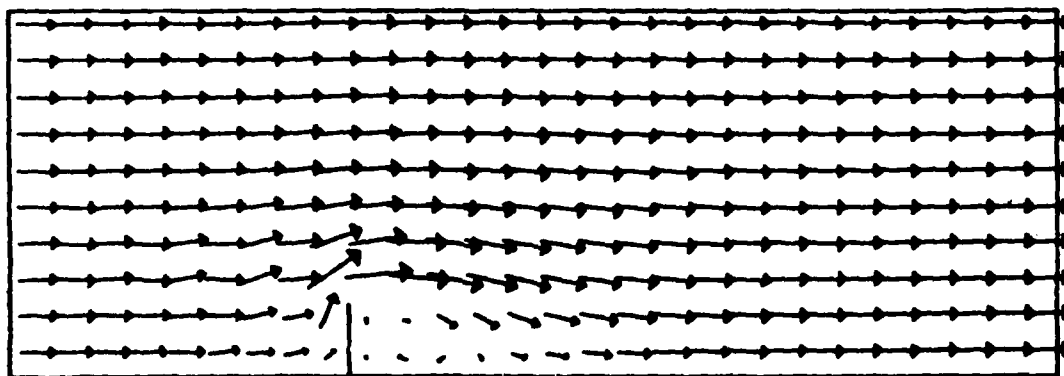


Figure 11. Flume barrier test with short barrier for assumption A2

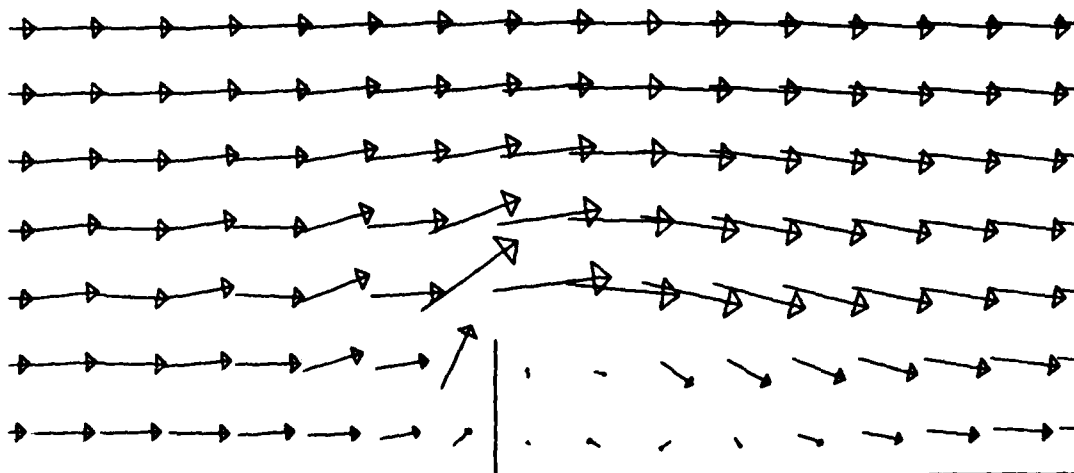


Figure 12. Enlargement of Figure 11 in area of barrier



### Surge Applications

18. Since WIFM capabilities were extended to include modeling of the continental shelf a comparison study between it and a recognized open-coast surge model was performed. The basis for the comparison was the problem of hindcasting water elevations caused by Hurricane Betsy (September 1965). SSURGE, an open-coast model by Wanstrath (1978), was previously applied to this problem in a study of coastal flooding for Louisiana. The windfield model used was a slight modification of an analytical model formulated by Jelesnianski (1965). All windfield parameters from the previous study were retained for use in the present comparison effort. Land effects on the windfield were not simulated. Figures 13 and 14 depict the Wanstrath and WIFM grids used. Figures 15 and 16 display hydrograph results at two representative gage locations. SSURGE does not account for flooding but does permit ponding areas through a finite height barrier coast. The grid extended only to the 300-ft contour. The WIFM grid extends to depth contours greater than 2,000 ft and treats the two-dimensional overland flooding and hence should be able to portray the surge more accurately. The mean absolute error in predicting open-coast observed peak watersurface elevations was essentially the same for both models (approximately 0.7 ft).

19. The WIFM grid shown in Figure 14 was devised especially for the comparison study. A grid, extending over a larger coastline reach and containing higher resolution (shown in Figure 17), was constructed for use in an ongoing study for the U. S. Army Engineer District, New Orleans. Simulation of Betsy was repeated but this time a version of the Standard Project Hurricane (SPH) parametric windfield model was used, one which introduces deformations to the windfield to account for land effects. This model of the coast was run with and without the advective and horizontal diffusion terms. Effects (on surface elevations) of including non-linear terms in the computations were minimal throughout the model region. The only noticeable effect was in the coastal area near landfall (Grand Isle, LA) where surface elevations were 0.3 to 0.6 ft higher.



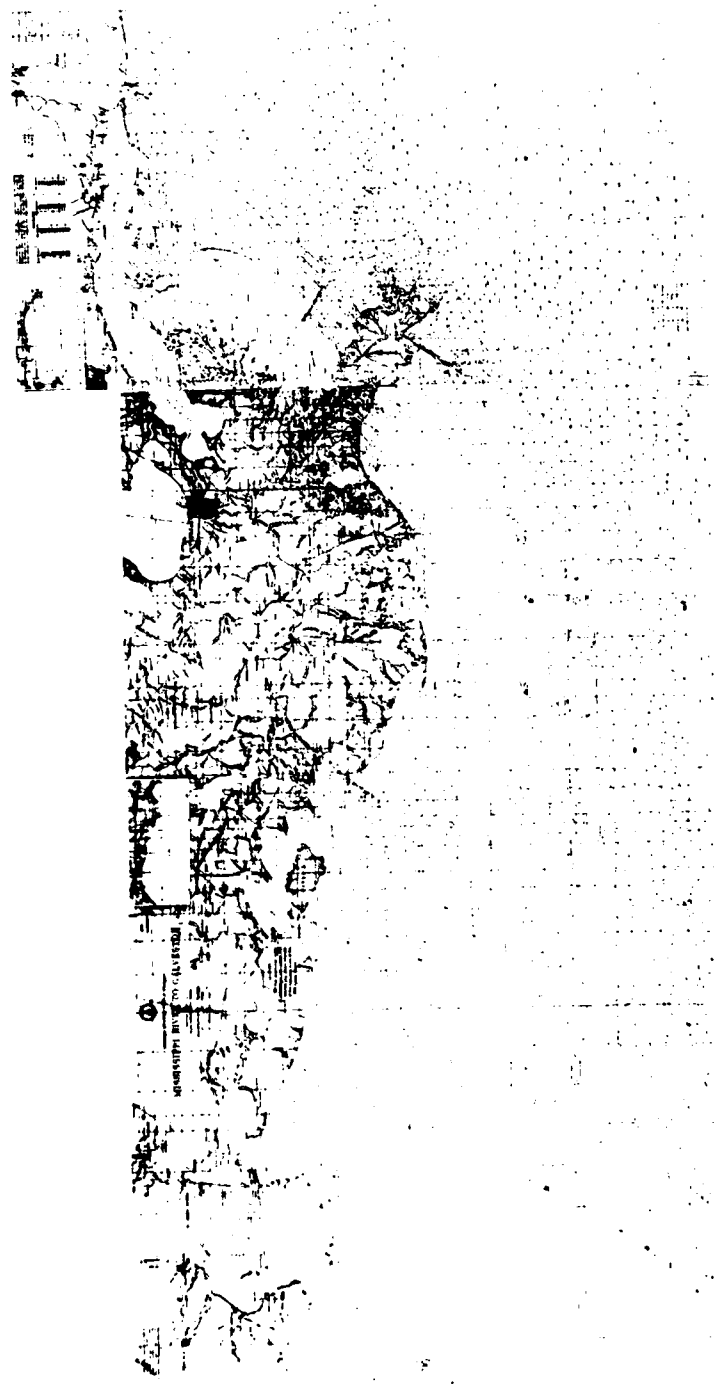


Figure 14. WIFM computational grid for simulating Hurricane Betsy to compare with SSURGE results

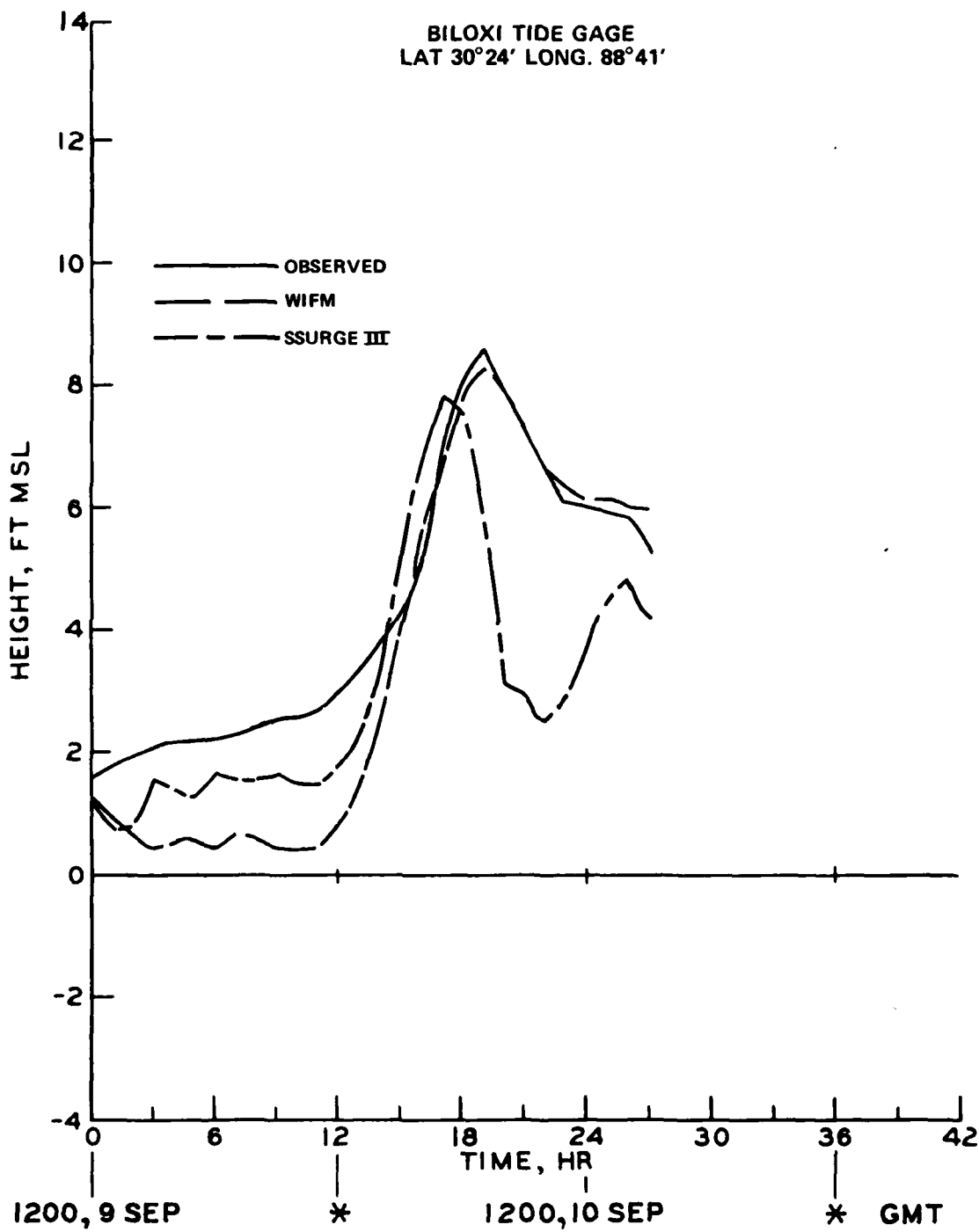


Figure 15. Comparison of WIFM and SSURGE hydrographs with observed data at Biloxi, MS

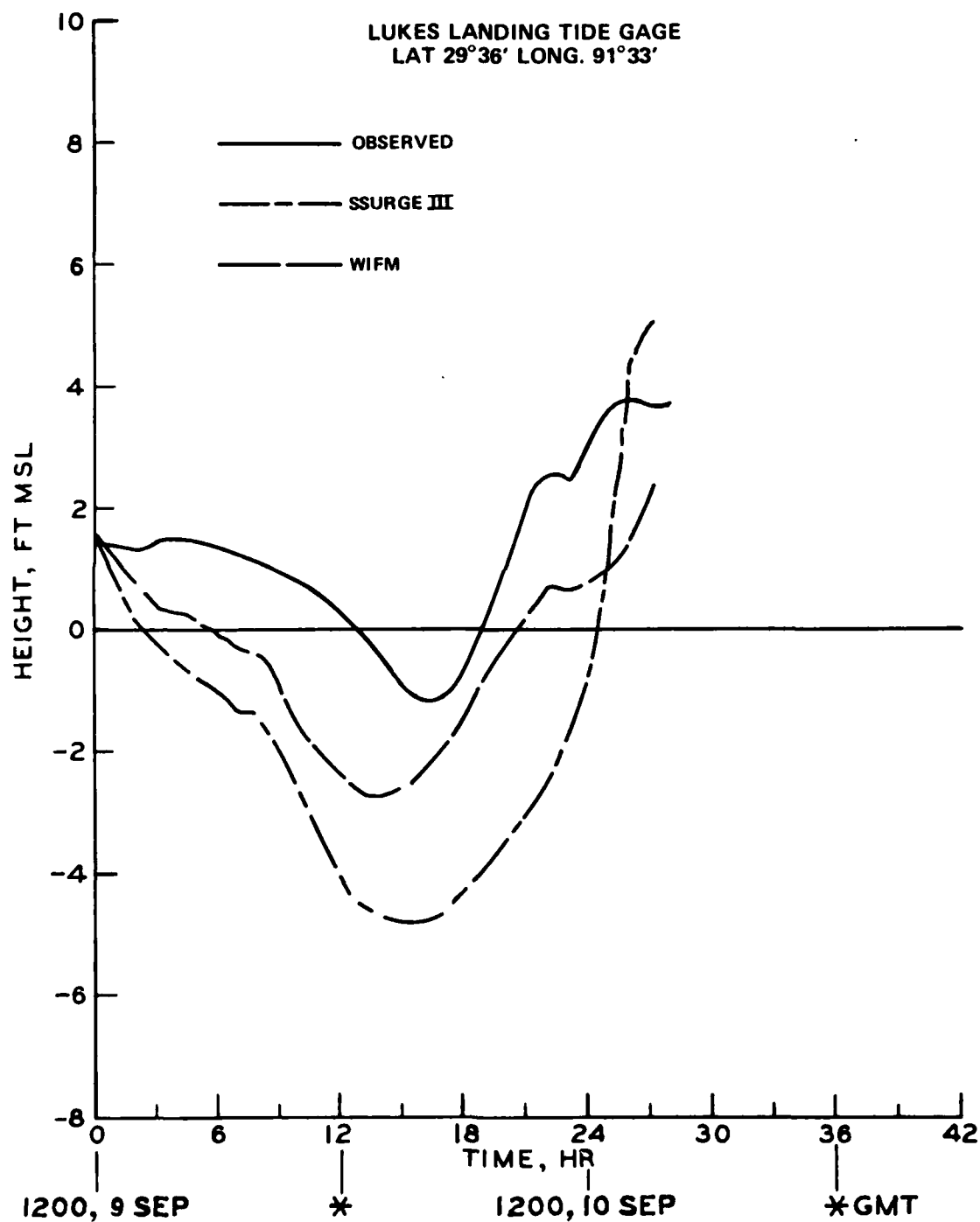


Figure 16. Comparison of WIFM and SSURGE hydrographs with observed data at Eugene Island, LA

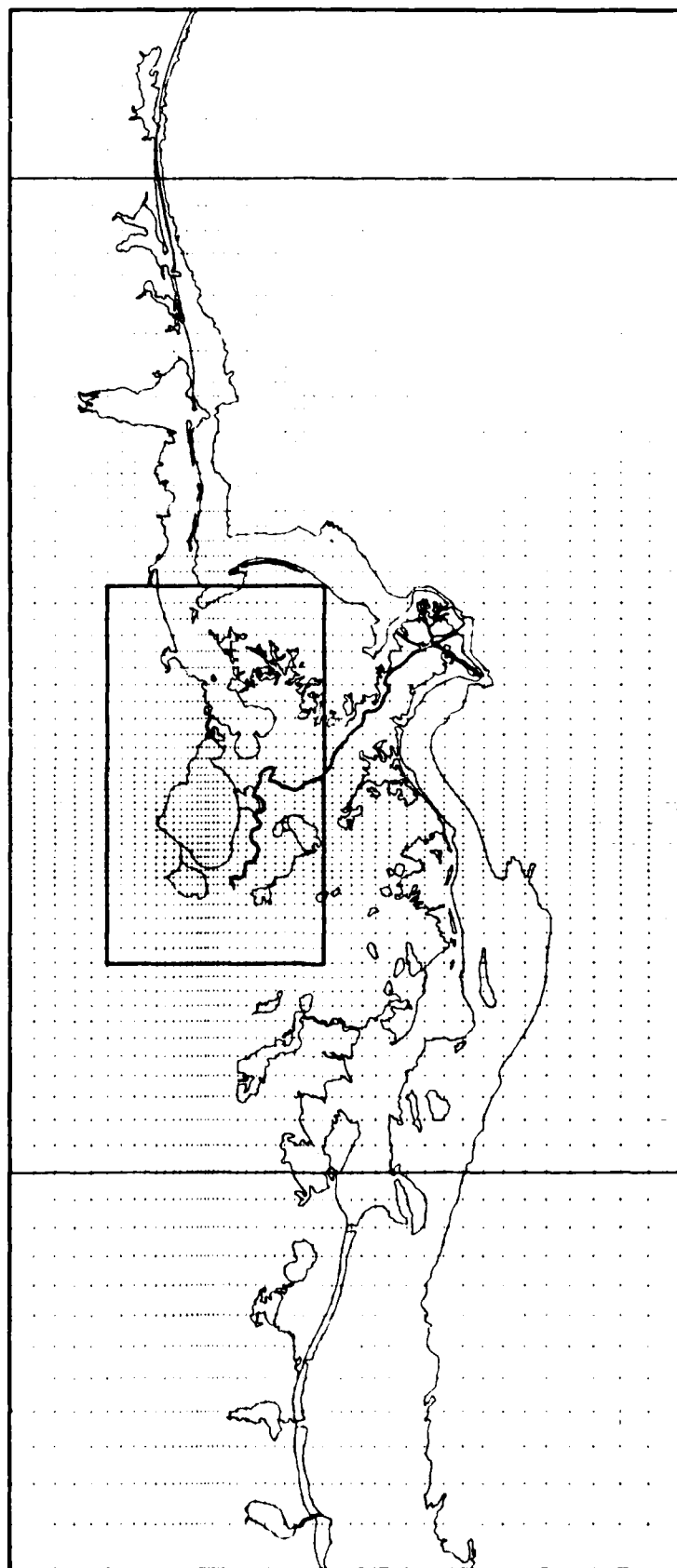


Figure 17. WIFM open-coast computational grid for the Lake Pontchartrain Hurricane Barrier Study

This increase in water level was a result of stronger non-linear term influence on circulation. Surface elevations generally increase near the tip of the Mississippi River Delta (to the right of landfall). Figure 18 depicts elevation and velocity comparisons for a gage located at the tip of the Delta. Changes in velocity magnitude and direction at this gage are typical of near-coast circulation differences.

20. Discharge ranges were established throughout the model area. A 2% reduction of transport through the major artery to Lake Pontchartrain was noted. Most other range comparisons showed the same minimal effect. The comparison for a range aligned north-south across the eastern end of the lake exhibited the largest difference in discharge (25% more in the ebb direction with the non-linear terms included). Since the range is long (about 8 miles) and circulation is unrestricted the effect of the non-linear terms on circulation is expected to be more significant.

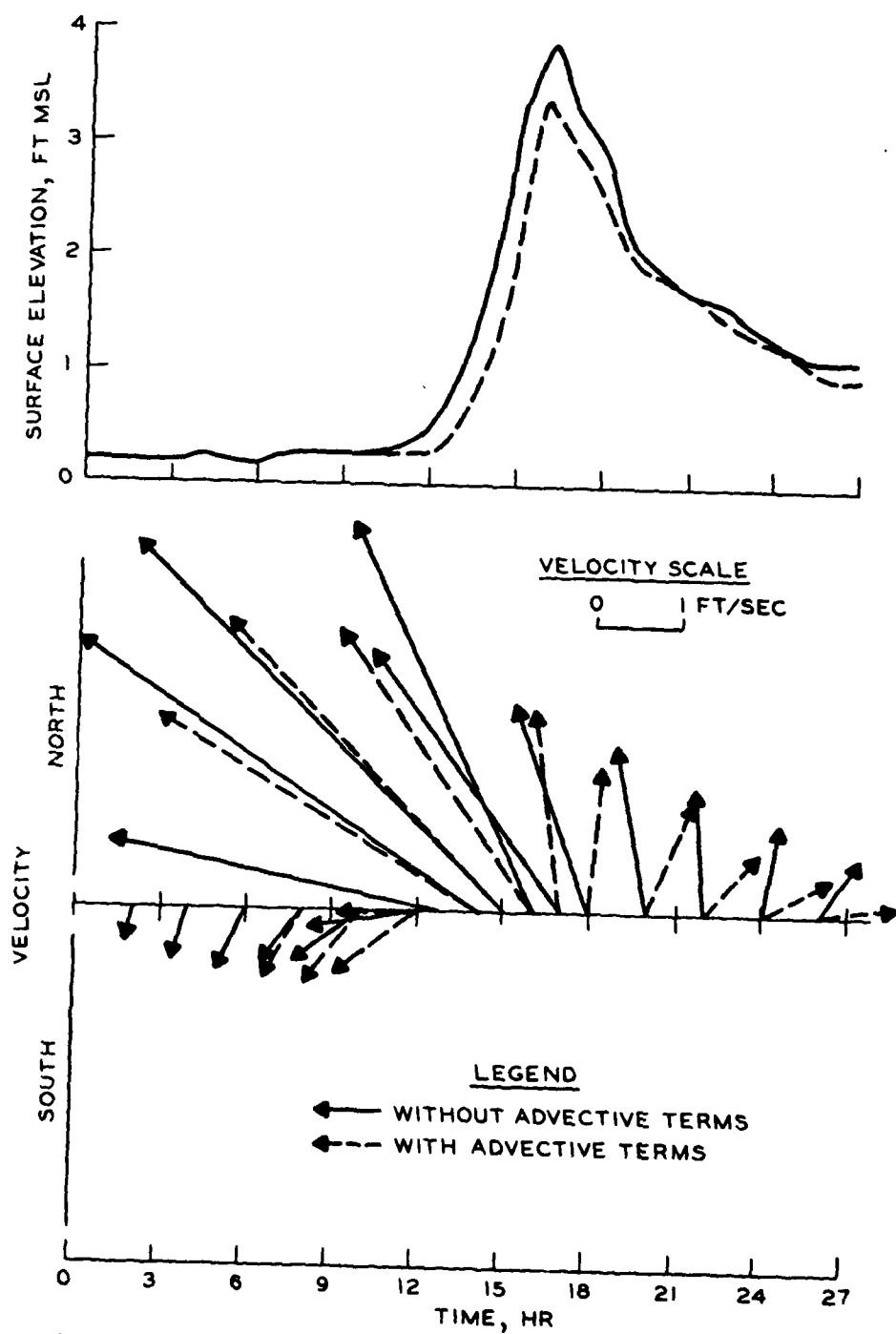


Figure 18. Surface elevation and velocity comparisons with and without the non-linear terms for an open-coast gage



#### PART IV: CONCLUSIONS AND RECOMMENDATIONS

21. An accurate method for representing the non-linear terms in the equations of motion was developed. No numerical stability problems were encountered while running the various tests described in this report. Additional tests for varied difference approximations of these terms near land-water boundaries are recommended for future research. In particular, tests of free-slip and no-slip representations are suggested. An attempt was made in this study to look at this problem but no definitive conclusions were reached. No-slip representations tested indicated much too severe of an effect on the current magnitudes in the cell adjacent to the boundary. WIFM was extended to simulate open-coast surge on the continental shelf and a successful comparison for Hurricane Betsy to a recognized open-coast model, SSURGE, was made. This development permits shelf and overland surge simulation on the same grid substantially reducing computational costs in future applications.

22. Research funding did not permit testing for advective effects in surge computations over a wide variety of conditions, both meteorological and varied land configurations. Catastrophic flooding from Betsy was simulated with and without the non-linear terms. Results from these runs showed that minimal effects on water-surface elevations and current magnitudes were caused by including the non-linear terms. The largest effect was on the peak surge at the coast near landfall (increases up to 0.6 ft) and circulation near the coast. The total discharge through the major lake entrance, The Rigolets, was decreased by a little over 2%. Based on this study the non-linear terms could be neglected if interests were solely water surface elevations. On the other hand, if current patterns were required (for example, to move sediment, pollutants, etc.) the results indicate that these terms should be included. Further testing is recommended in future U. S. Army surge investigations.

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